

## **Chapter 7: Analysis of Data from the Fisher Deep Mine Site, Lycoming County, PA**

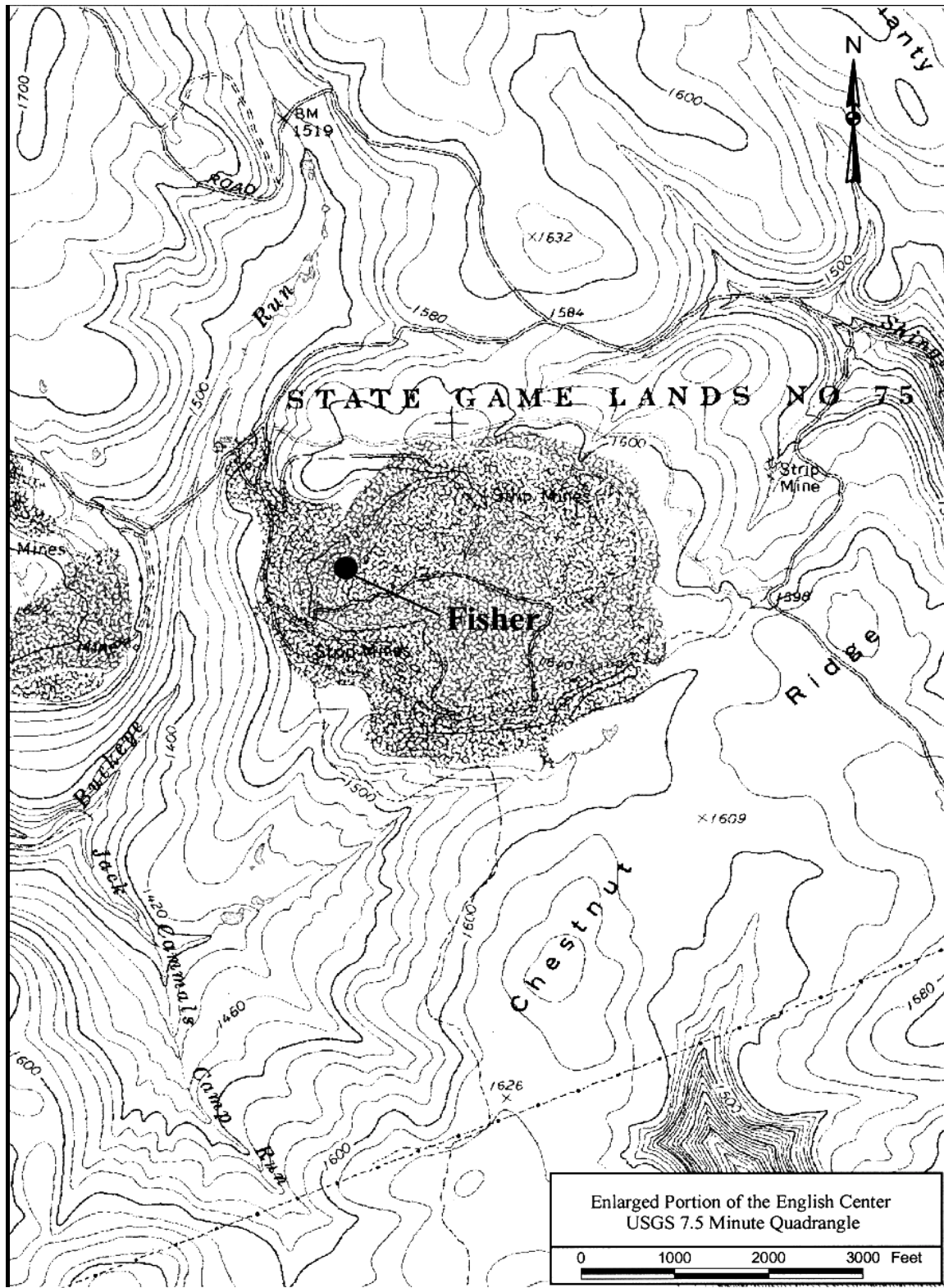
The Fisher site is located in Lycoming County, Pennsylvania near the village of English Center (Figure 7.0). Prior to remining on this site, the land surface was extensively disturbed by abandoned mine pits and spoil piles, and the Fisher deep mine, a large abandoned underground mine, occupied much of the subsurface. Fisher deep mine discharge (monitoring point M-1) characteristics have been discussed in numerous other reports including Section 5 of EPA's *Coal Remining Statistical Support Document* (EPA-821-B-00-001).

The Fisher deep mine discharge and its impact on the receiving streams is discussed in an Operation Scarlift Report of 1977 on the Little Pine Creek Watershed. The Buckeye Run and Otter Run tributaries of the Little Pine Creek were impacted by AMD from the Fisher deep mine. Otter Run was a prolific native brook trout stream prior to being impacted by the Fisher deep mine discharge, and it has returned to a trout fishery as the result of remining operations. Descriptions of the remining operation, geologic characteristics of the area and water quality improvements are included in Plowman (1989) and Smith and Dodge (1995).

The data set that was analyzed statistically in this chapter (see report by Dr. J.C. Griffiths, December 1987) includes all baseline pollution load data (i.e., prior to issuance of the first remining permit) and data from the first year and a half of remining. Baseline pollution load data collection took place from May/June 1982 through 1985. The primary remining permit was issued on November 5, 1985, and remining operations commenced by February 1986. Final coal removal occurred on June 1995 and backfilling was essentially completed within that permit area by February 1996. The primary remining permit for this site is contiguous to a previous permit that did not involve daylighting and to a subsequent remining permit that was issued in 1994 and completed in July 1999 (that also drained to the M-1 discharge). The total acreage of these three permits is 542, of which approximately 200 acres were mined under the initial permit (issued prior to 1985). The data set included in Section 5 of the EPA *Coal Remining Statistical Support Document* includes monitoring data for the M-1 discharge from 1981 to 1998. Time plots and box plots of net acidity, acid load, iron load and net alkalinity show changes in water quality and pollution load over the four year baseline period, ten years of remining, and two years following the completion of backfilling of the remining site.

The data analysis presented in this chapter follows the usual flow diagram (Figure 3.1). The data consist of 79 observations of seven parameters. Flow measurements began on June 9, 1982 and remining of the site began on February 4, 1986. There were three observations prior to June 9, 1982 (see Appendix E Table). After excluding these observations and inserting mean values for samples with a missing parameter, 57 observations remained prior to remining and 19 observations remained after remining commenced. From the histograms showing skewness of varying degrees, it was decided to log-transform (base ten) the data.

Figure 7.0: Map of Fisher Mine Site



After preliminary analysis of the data, the bivariate and time series plots appeared to be somewhat irregular, and it was decided to measure the intervals between the observations by creating a new variable (the first differences between the number of days).

The intervals between observations (days) vary from extremes of one to 104, with a mean of 26.7 days. This mean is nearly equal to the median (26.5), indicating that the frequency distribution is symmetrical. The central part of the distribution  $Q_1$  to  $Q_3$  lies between 12.7 and 33 days. The most serious discrepancies are, however, that there are five observations between 70 and 104 days, and four of these are 90 days or more. These large gaps in the data preclude rigorous time series analysis which requires approximately equal intervals between observations.

## Univariate Analysis

The coefficient of variation (CV%) for flow, acidity, sulfate and manganese (Table 7.1) are all less than or equal to 20%. This is surprisingly low when compared to previous data analysis. Iron, however, possesses a coefficient of variation of 929 % and aluminum also has a large CV (71%).

**Table 7.1: Summary Statistics for 79 Log Transformed Observations**

	N	N*	Mean	Median	Trimmed Mean	Standard Deviation	Standard Error of the Mean
Flow	79	0	1.7882	1.8062	1.7754	0.3734	0.0420
Acidity	79	0	1.8700	1.8274	1.8660	0.2183	0.0246
SO <sub>4</sub>	79	0	2.5316	2.5105	2.5342	0.2124	0.0239
Total Iron	79	0	0.0442	0.0825	0.0575	0.4106	0.0462
Mn	79	0	0.9396	0.9513	0.9335	0.1716	0.0193
Al	79	0	0.4959	0.5539	0.5029	0.3539	0.0398
Interval	78	1	26.72	26.50	24.20	19.95	2.26

	Minimum	Maximum	First Quartile	Third Quartile	Coefficient of Variation
Flow	0.9542	2.7882	1.4771	2.0000	20.9
Acidity	1.4409	2.3747	1.7076	2.0453	11.7
SO <sub>4</sub>	1.6902	3.0792	2.4346	2.6335	8.4
Total Iron	-1.301	0.8450	-0.1024	0.2032	929.3
Mn	0.5775	1.5185	0.8500	1.0253	18.3
Al	-0.4948	1.4698	0.3598	0.6628	71.4
Interval	1.00	104.00	12.75	33.00	

There is little doubt that the coefficient of variation for iron is misleading and serves to illustrate one of the dangers of using the CV%. When the mean is very small, as in this case, the CV tends to become very large, particularly in ratio-type data (i.e., percent or concentration, Griffiths, 1967, Chapter 15, page 316). It should be used on log data with great care, if at all.

When the data are subdivided into 57 observations (from the beginning of flow measurement to immediately prior to remining, Table 7.2a), and into 19 observations (after commencement of remining, Table 7.2b), the CVs of flow, acidity, sulfate, and manganese remain substantially similar. Iron, however, shows a marked drop from a CV equal to 109.2 % to a CV equal to 50.2%, implying that there was a major change in variability after the start of remining. The CV of aluminum, on the other hand, shows no change from the original data set.

**Table 7.2a: Summary Statistics for 57 Log Transformed Observations (Pre-remining)**

	N	Mean	Median	Trimmed Mean	Standard Deviation	Standard Error of the Mean
Flow	57	1.7885	1.8062	1.7751	0.3793	0.0502
Acidity	57	1.9176	1.9222	1.9170	0.1964	0.0260
SO <sub>4</sub>	57	2.4654	2.4771	2.4744	0.1881	0.0249
Total Iron	57	0.2027	0.1461	0.1961	0.2214	0.0293
Mn	57	0.9661	0.9713	0.9652	0.1302	0.0172
Al	57	0.4874	0.5250	0.4928	0.3520	0.0466

	Minimum	Maximum	First Quartile	Third Quartile	Coefficient of Variation
Flow	0.9542	2.7882	1.4771	2.000	21.2
Acidity	1.4564	2.3747	1.7489	2.0737	10.2
SO <sub>4</sub>	1.6902	3.0792	2.4013	2.5682	7.6
Total Iron	-0.1427	0.7243	0.0453	0.3444	109.2
Mn	0.5775	1.4048	0.8836	1.0528	13.5
Al	-0.4948	1.4698	0.3874	0.6389	72.2

**Table 7.2b: Summary Statistics for 19 Log Transformed Observations (During remining)**

	N	Mean	Median	Trimmed Mean	Standard Deviation	Standard Error of the Mean
Flow	19	1.7702	1.8062	1.7551	0.3867	0.0887
Acidity	19	1.6673	1.6928	1.6733	0.1018	0.0234
SO <sub>4</sub>	19	2.6844	2.6335	2.6830	0.1695	0.0389
Total Iron	19	-0.5345	-0.5376	-0.5170	0.2684	0.0616
Mn	19	0.7988	0.7672	0.7953	0.1430	0.0328
Al	19	0.4865	0.6542	0.4954	0.3818	0.0876

	Minimum	Maximum	First Quartile	Third Quartile	Coefficient of Variation
Flow	1.1461	2.6513	1.4771	2.0000	21.8
Acidity	1.4409	1.7924	1.5694	1.7543	6.1
SO <sub>4</sub>	2.4150	2.9777	2.5658	2.8751	6.3
Total Iron	-1.3010	-0.0655	-0.6021	-0.3565	50.2
Mn	0.6010	1.0569	0.6656	0.9101	17.9
Al	-0.3010	1.1239	0.2504	0.7627	78.5

The means also show interesting changes. Acidity possesses an overall mean of 1.87. In comparison, the mean of acidity prior to remining (1.92) is larger than during remining (e.g., see Figure 2.5, Chapter 2). Sulfate is lower than the overall mean prior to remining and much higher than the overall mean during remining. Log iron shows the most substantial change, from 0.20 before remining (approximately 1.6 in untransformed data units) to -0.53 (0.295) after after remining operations began. This represents a very large and favorable change because the pollution load has been reduced. Manganese also shows a quite large change from before to during remining.

The histograms of log transformed flow (Figure 7.1a), acidity (Figure 7.1b), iron (Figure 7.1c), manganese (Figure 7.1d), and aluminum (Figure 7.1e) are essentially symmetrical, thus the transformation has sufficed to reduce the asymmetry in the original data. Because of the gaps in the data and their peculiar pattern of variation, it was decided to graph some of the parameters to show the distribution of gaps and to examine the pattern for cycles.

Figure 7.1a: Histogram of Log Flow

Histogram of FLOW    N = 79

Midpoint	Count	
1.0	1	*
1.2	8	*****
1.4	12	*****
1.6	11	*****
1.8	17	*****
2.0	17	*****
2.2	5	*****
2.4	5	*****
2.6	2	**
2.8	1	*

Figure 7.1b: Histogram of Log Acid

Histogram of ACID    N = 79

Midpoint	Count	
1.4	1	*
1.5	4	****
1.6	5	*****
1.7	18	*****
1.8	14	*****
1.9	9	*****
2.0	10	*****
2.1	9	*****
2.2	5	*****
2.3	3	***
2.4	1	*

Figure 7.1c: Histogram of Log Iron

Histogram of FE    N=79

Midpoint	Count	
-1.2	1	*
-1.0	1	*
-0.8	1	*
-0.6	8	*****
-0.4	6	*****
-0.2	3	***
0.0	22	*****
0.2	17	*****
0.4	9	*****
0.6	9	*****
0.8	2	**

Figure 7.1d: Histogram of Log Manganese

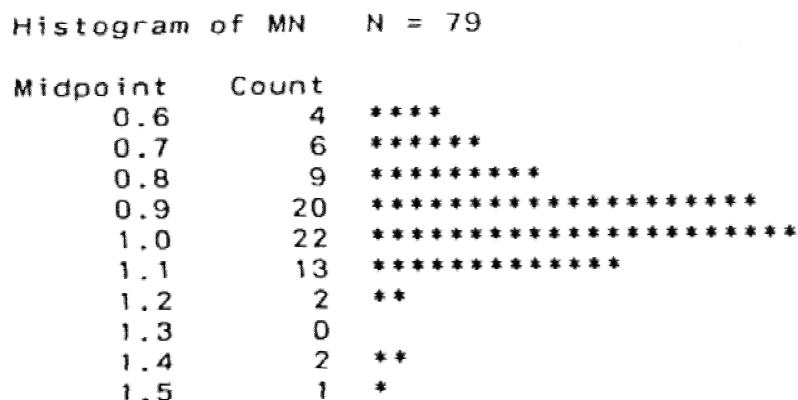
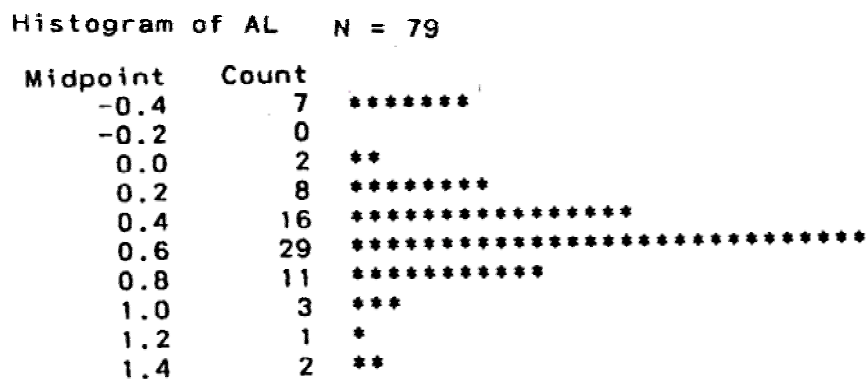


Figure 7.1e: Histogram of Log Aluminum



Log flow versus days is shown in Figure 7.2a, and there does not appear to be much change coincident with the start of remining. Furthermore, peak flows occur in various months throughout the record; there are two in June (1982, 1985) and two in April (1984, 1987) for example, but they do not appear to recur each year. No persistent cyclic pattern is evident for flow.

Log acidity (Figure 7.2b) shows a large change, remaining well above both the mean and median from November 27, 1981 to September 6, 1984, then falling to the mean around December 18, 1984 and falling consistently below both mean and median following October 26, 1985. This change took place prior to activation of the remining permit. However, mining was occurring on an adjacent surface mine prior to 1985. The mean prior to remining (N = 57, Table 7.2a), is 1.9176 (log transformed) and is 82.7 in untransformed units. After remining, the mean is 1.6673 in log-transformed units and 46.5 in untransformed units. When the quality control limits around

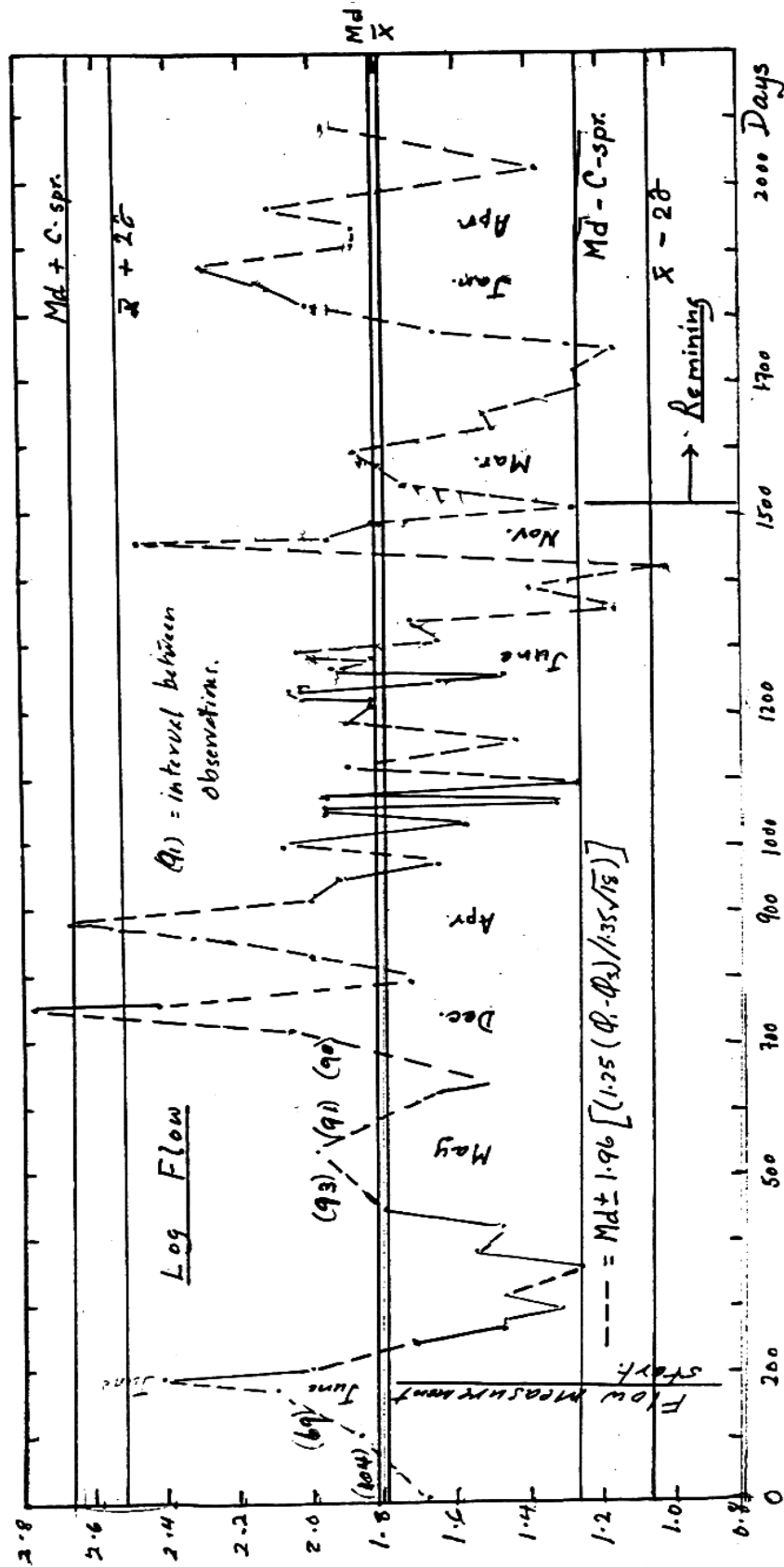
the median are inserted using a sample size of 18, the post-remining median is significantly below the pre-remining limits (see lines in Figure 7.2b).

Graphs of log iron and log manganese are included in Figures 7.2c and 7.2d respectively. Log iron shows a marked decrease over time during the pre-remining period with a sharp decline immediately following commencement of remining. The confidence limits around the median of pre-remining are inserted in the graph. The median and the confidence limits, after remining began, are much lower and the median lies outside the confidence limits of pre-mining.

Log manganese also shows a decrease after remining began but is not nearly so marked as is log iron. However, as in the case of log iron, the median log manganese after remining remains outside (below) the pre-remining confidence limits. Log aluminum is plotted against days (Figure 7.2e), and the pre- and post-remining statistics are fairly similar. The post-remining median lies within the confidence limits (for  $N' = 18$ ) of the pre-remining performance. There is no substantial change in the central tendency.



Figure 7.2a: Log Flow vs. Time (Days)



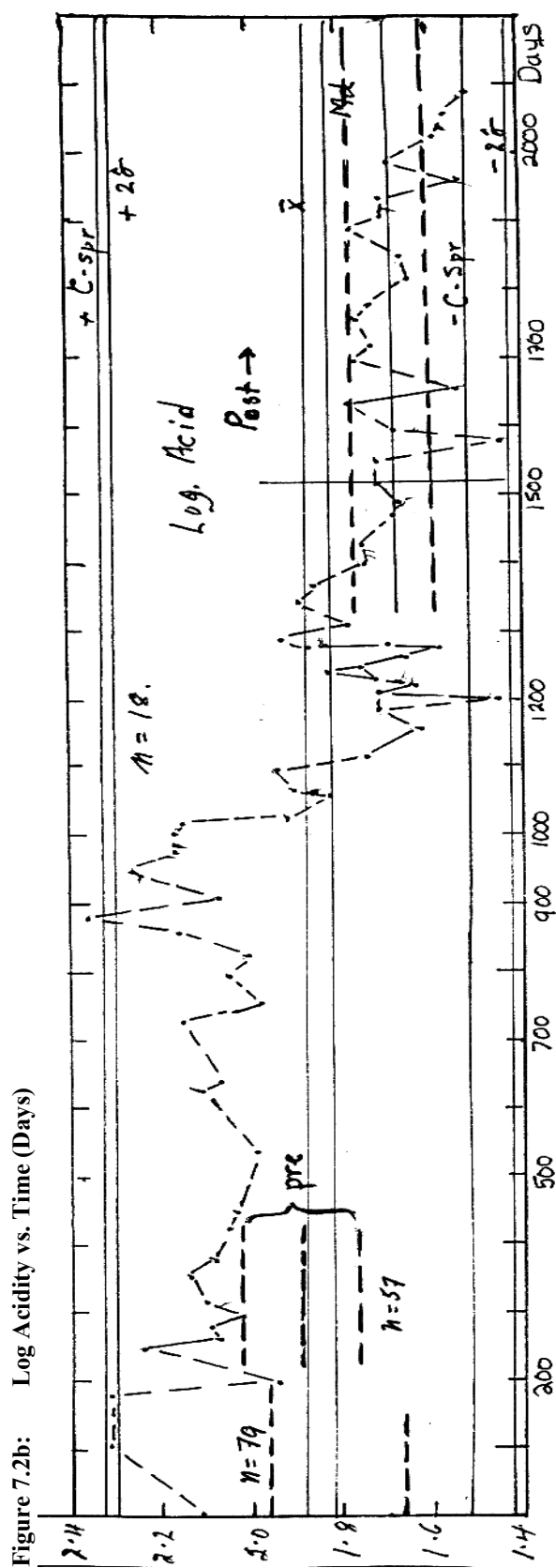


Figure 7.2c: Log Iron vs. Time (Days)

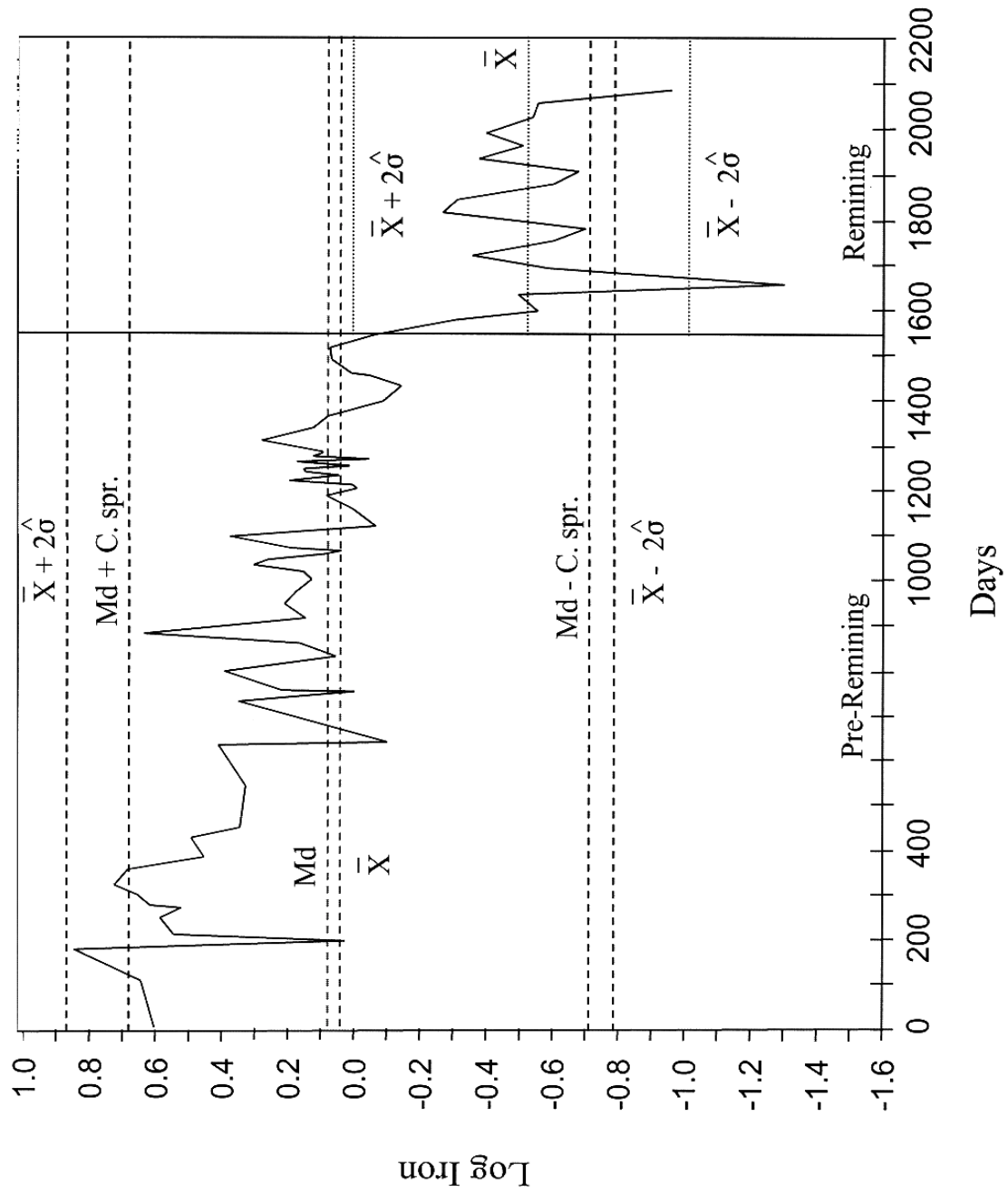


Figure 7.2d: Log Manganese vs. Time (Days)

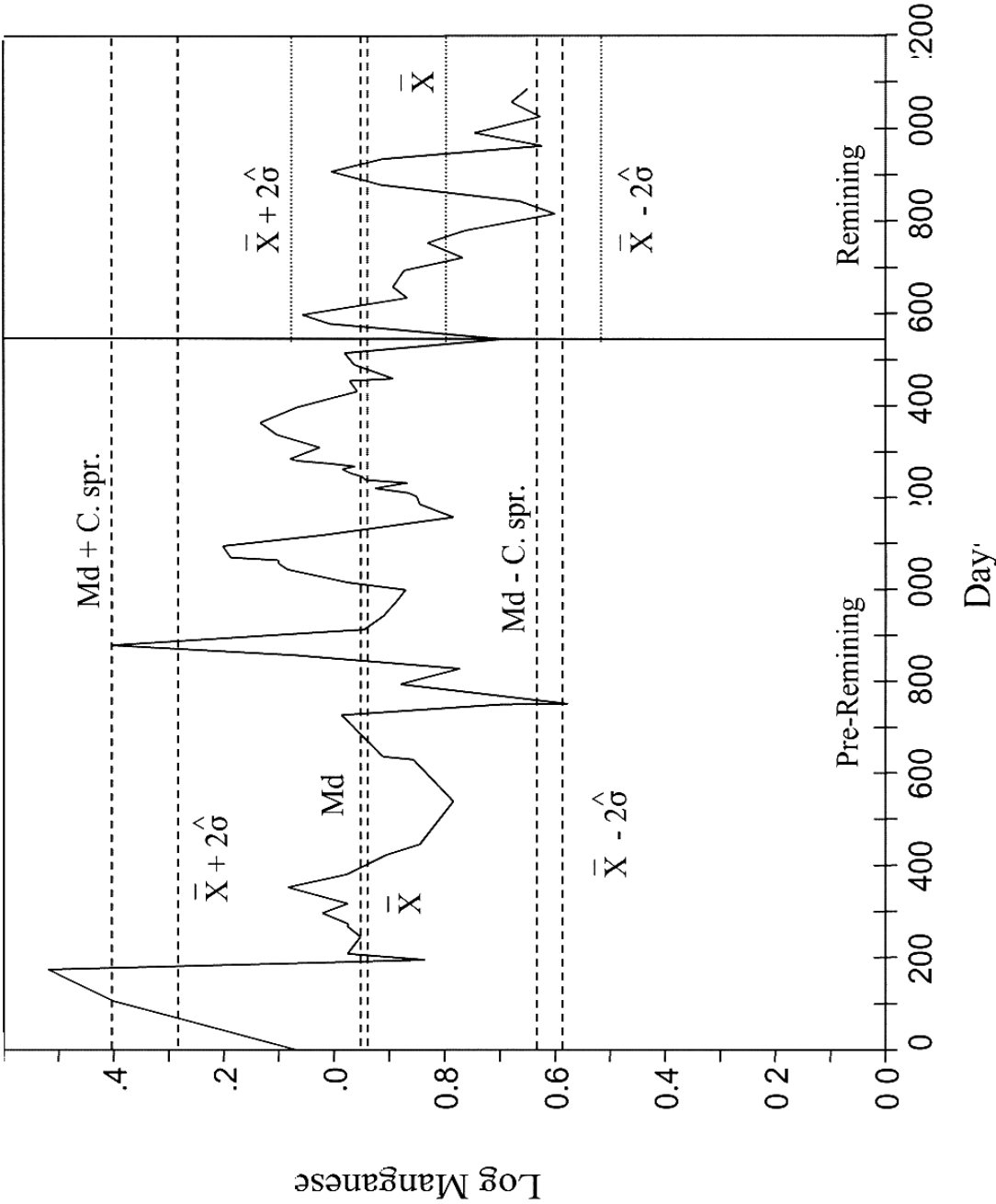
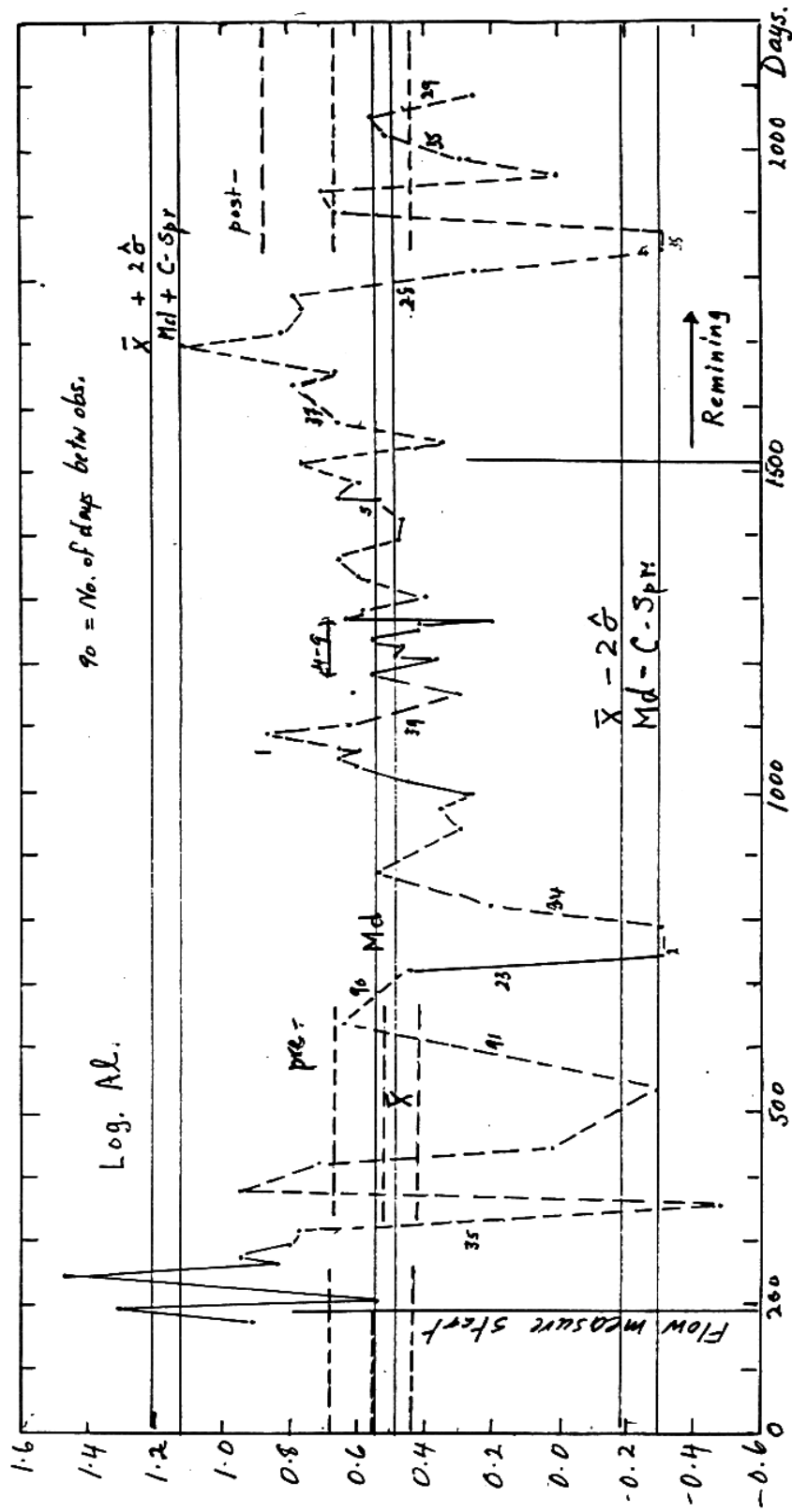


Figure 7.2e: Log Aluminum vs. Time (Days)



Another comparison of interest between pre- and post-remining water quality concerns variances, and parameter pairs may be compared by a variance ratio or F-test. It is customary to divide the smaller variance into the larger so that the outcome always equals or exceeds one. This is in accord with the F-table of values which tests only that half of the F distribution that exceeds unity. In this case, it begins with the flow variables as follows: the calculated ratio,  $F_{\text{calc}} = 0.1495/0.1439 = 1.039$ . The expected F-value ( $F_{18,55} = 1.79$ , at the five percent level) is much larger, thus, there is no difference in the variances before and after remining.

In the variance ratio for acidity,  $F_{\text{calc}}$  equals  $0.0386/0.0104 = 3.722$ . The five percent level of expected F for these degrees of freedom (df) is  $F_{50,18} = 2.02$ , so that the variance for pre-remining is significantly larger than that for post-remining (a desirable outcome). The same test performed for  $\text{SO}_4$  yields  $F_{\text{calc}}$  equal to  $0.0354/0.0287 = 1.232$ . Thus, the pre-remining variance is not significantly different from that after remining.

For iron,  $F_{\text{calc}}$  equals  $0.0720/0.0490 = 1.469$  (not significantly greater than a 5% probability level of  $F_{18,56} = 1.79$ ). The variance of iron concentration after remining is not significantly different from the variance of iron concentration before remining. Manganese has a slightly larger variance post- than pre-remining yielding an  $F_{\text{calc}} = 0.020/0.017 = 1.206$ , but the difference in variability is not statistically significant. Aluminum also possesses a larger variance after remining than before; the corresponding  $F_{\text{calc}} = 0.1458/0.1239 = 1.176$ . This difference is not significant.

The behavior of the variance before and after the beginning of remining is as important as the differences in mean or median. This is because if the variance becomes significantly smaller after remining begins, the observed value is much less likely to exceed the confidence limits at some future time, assuming that the behavior remains consistent.

## **Bivariate Analysis**

Examination of the relationship among pairs of variables begins with the correlations of zero order among all pairs of the seven variables (Table 7.3). The expected value of the correlation coefficient for a pair of variables with 79 observations (= 78 df), each taken from a population in which there is no correlation, is approximately  $r = 0.217$  (using 80 df from Table 21, Arkin and Colton, 1950, p. 140, Table of r for the 1% and 5% points of the r distribution). This means that any  $r < 0.217$  is not significantly different from zero. This value is found at the top of Table 7.3.

**Table 7.3: Correlations of Zero Order Among the Seven Variables**

Correlation of Seven Variables						
	Days	Flow	Acidity	SO <sub>4</sub>	Total Iron	Mn
Flow	-0.044					
Acidity	-0.783	0.082				
SO <sub>4</sub>	0.199	-0.006	-0.093			
Total Iron	-0.852	-0.013	0.724	-0.211		
Mn	-0.410	-0.042	0.457	0.396	0.510	
Al	-0.121	-0.307	0.058	0.250	0.055	0.361

Pairing each variable in turn, against days, shows that a linear association between flow, sulfate, or aluminum and days is unlikely. The relationship between acidity and days is negative (i.e., inverse). Acidity decreases as days increase. This is also the case with the relationship between days and iron. In both cases, the proportion of common association ( $r$ ) among the pairs of variables is large, 61% for acidity and 73% for iron. Manganese also shows an inverse relationship with days but the degree of association is much less ( $r^2 = 17\%$ ).

Relationships between flow and the other variables, as measured by the correlation coefficient, is effectively zero. The exception is aluminum, where the relationship is negative (inverse) and the degree of association is not very strong ( $r^2 = 9\%$ ).

Acidity appears to have no simple linear relationship with either sulfate or aluminum, however, it is positively associated with iron, possessing an  $r^2 = 52\%$  in common. Acidity has  $r^2 = 21\%$  common association with manganese and again the relationship is positive (i.e., they increase or decrease together). Sulfate and manganese are positively associated but the degree of common association is weak. Variation in manganese is related to variation in iron in the same way but to a slightly greater degree. Manganese is also weakly positively associated with aluminum (the degree of common association  $r^2 = 13\%$ ). The strongest correlation coefficient values are between the pairs of acidity and days, and iron and days. The decline of acidity and iron with time is obvious in Figures 7.2b and 7.2c. It is no surprise, therefore, that the third strongest association is the positive one between iron and acidity.

As a check that perhaps the maximum degree of association was not of zero order, the cross-correlation functions (CCF) were run and the principal outcomes are listed in Table 7.4. To evaluate these functions, it is reasonable to take a conservative value (for example,  $r = 0.3$ ) as the limit below which the relationship is not significantly different from zero. In the case of flow versus the other variables, there appears to be no linear association except for aluminum which has its highest value as inverse ( $-0.389$ ) at lag  $-4$ . It is likely that the zero order value of  $-0.302$  is not really significantly different from the  $r$  value at lag  $-4$ .

**Table 7.4: Summary of Important Cross-correlation Functions (CCF) Among Seven Variables**

	Variables	$r_{\max}$	lag @ $r_{\max}$	$ r  > 0.3$
1.	Flow vs. Acid	0.248	- 16	none
2.	Flow vs. $\text{SO}_4$	- 0.254	- 4	none
3.	Flow vs. Fe	0.214	10	none
4.	Flow vs. Mn	- 0.276	- 4	none
5.	Flow vs. Al	- 0.389	- 4	-4, -3, 0
6.	Acid vs. $\text{SO}_4$	- 0.367	14	2 to 4, 14, 15
7.	Acid vs. Fe	0.724	0	- 14 to 14, 16 to 18
8.	Acid vs. Mn	0.457	0	0, 1
9.	Acid vs. Al	0.252	- 18	none
10.	$\text{SO}_4$ vs. Fe	- 0.361	-7	- 9 to - 6, - 4, 2
11.	$\text{SO}_4$ vs. Mn	0.396	0	0, 3 to 5
12.	$\text{SO}_4$ vs. Al	0.299	1	none
13.	Fe vs. Mn	0.511	0	-2 to 3
14.	Fe vs. Al	- 0.188	- 12	none
15.	Mn vs. Al	0.441	1	- 13, 0, 1
16.	Acidity vs. days	- 0.783	0	- 18 to 17
17.	Iron vs days	- 0.852	0	-17, - 15 to 13

Acidity versus sulfate, iron, and manganese are all larger than the critical value of 0.3. The cross-correlation function for sulfate has three values exceeding 0.3 (at lags of +4, +2 and +14). However, these values are all indicative of a low degree of association (<13%) between the two variables. Iron and manganese achieve their maximum  $r$  at zero lag.

Sulfate versus iron, manganese, and aluminum show correlations between 0.3 (0.299) and 0.396. These values are all significantly different from zero. Correlations are positive between sulfate, manganese, and aluminum, but negative with iron. Sulfate behaves independently in all associations with other variables. The maximum correlation between iron and manganese (0.510) occurred at 0 lag, and indicates a relatively strong degree of association (26%). The relationship between iron and aluminum never exceeded the critical value of  $r = 0.217$ .

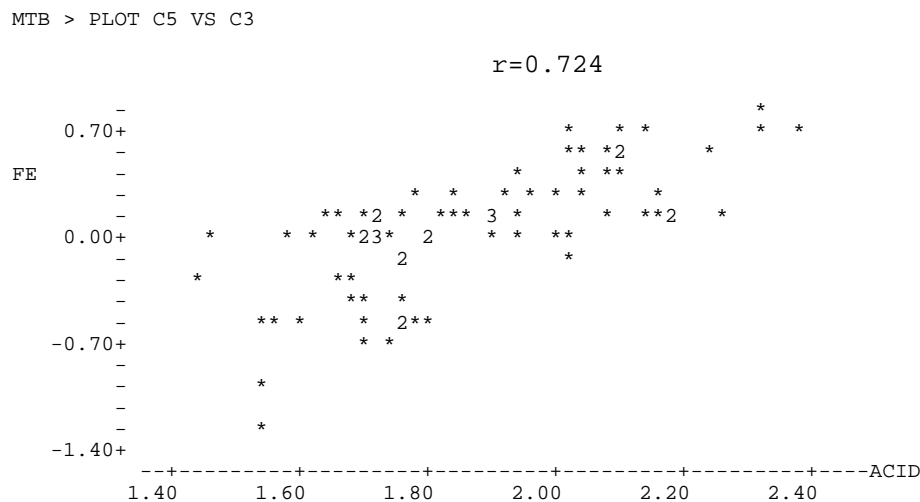
The correlation between iron and manganese is very weak (<26%) and the maximum is at zero lag. The relationship between iron and aluminum does not exceed the critical value of 0.217 (see Table 7.3).



The relationship of manganese and aluminum exceeds the critical value of 0.217 at lag -13 to -10, lag 0 to 1, and lag 17. The maximum  $r$  (0.441) occurs at lag 1. Again, if this is a real association, it is weak ( $r^2 = 19\%$ ).

Bivariate relationships between pairs of water quality parameters and flow vs. water quality parameters were plotted. The results yielded very little that was meaningful with the exception of iron vs. acidity (Figure 7.3). This relationship had the highest positive correlation coefficient ( $r = 0.724$ ).

**Figure 7.3: Plot of Log Iron vs. Acidity**



## Time Series Analysis

This analysis is subdivided into three parts. First, there is a plot of each variable versus the date collected. The dates are forced into 79 equal intervals, distorting the graph in terms of horizontal scale. (The correct spacing may be seen in Figures 7.2a to 7.2d.) The second subdivision details the diagnosis phase of the Box-Jenkins time series analysis using the autocorrelation and partial autocorrelation functions (Acf and Pacf, respectively). The third stage comprises modeling using Box-Jenkins estimation and forecasting programs.

The time series graphs begin with a plot using the variable of the first differences between collection dates against the observation number(s) (Figure 7.4a). The trend increases consistently through time. Figure 7.4a is included as an example of what happens when a variable of known structure is analyzed, where any variable with a constant function (increasing or decreasing) over time will yield a typical Acf and Pacf (Figures 7.5a and 7.5b).

Log flow is plotted against equal intervals in Figure 7.4b. The variation around the mean appears to remain reasonably constant throughout the period of observation. By contrast, in a plot of log acidity versus date (Figure 7.4c) the variation in acidity is consistently high and above

the mean until approximately the 29<sup>th</sup> observation (September 6, 1984), when it decreases to the mean from observation 30 to 36 (September 21, 1984 to November 23, 1984). The pattern of variation then falls well below the mean from observation 37 to 49 (December 18, 1984 to May 22, 1985); from observation 50 to 54 (May 28, 1985 to August 19, 1985) it remains close to the mean, and from 55 to 79 (September 21, 1985 to August 12, 1987) the range of acidity values remains consistently below the mean. Remining began at observation 61 (February 17, 1986).

Sulfate versus observation number (Figure 7.4d) shows no substantive change in variation around the mean throughout all 79 observations. The plot of iron versus observation number (Figure 7.4e) shows a slight decreasing trend prior to remining at observation 61. From observation 62 (March 22, 1986) onwards, the variation is well below the mean with two observations (65, June 10, 1986 and 89, August 12, 1987) below the lower two standard deviation limit.

Fluctuations in the concentrations of manganese (Figure 7.4f) are quite large, particularly in the beginning (observations 1 through 3, November 27, 1981 to May 19, 1982). From observation 4 through observation 66 (June 9, 1982 through July 15, 1986), the fluctuations are around the mean (= the median), and from observation 67 to 79 (August 12, 1986 to August 12, 1987), the observations tend to fall below the mean, varying widely, from observation 73 (February 14, 1987) slightly above the mean to observation 70 and observations 75 to 79 (November 15, 1986 and April 11, 1987 to August 12, 1987) near the lower confidence limits.

The time series plot of aluminum (Figure 7.4g) begins well above the mean (= the median) in observations 3 to 9 (May 19, 1982 to September 18, 1982), then falls well below the mean for observations 11 to 22 (November 13, 1982 to March 2, 1984). Observations 11, 15, and 19 to 21 (November 13, 1982, May 18, 1983 and December 15, 1983 to January 28, 1984) are all below the lower confidence limits. For observations 23 to 79 (March 31, 1984 to August 12, 1987), the concentration falls around the mean with two strong deviations to the lower confidence limits at observations 71 and 72 (December 13, 1986 and January 17, 1987). Remining does not seem to have had any consistent effect.



Figure 7.4c: Plot of Log Acidity vs. Time

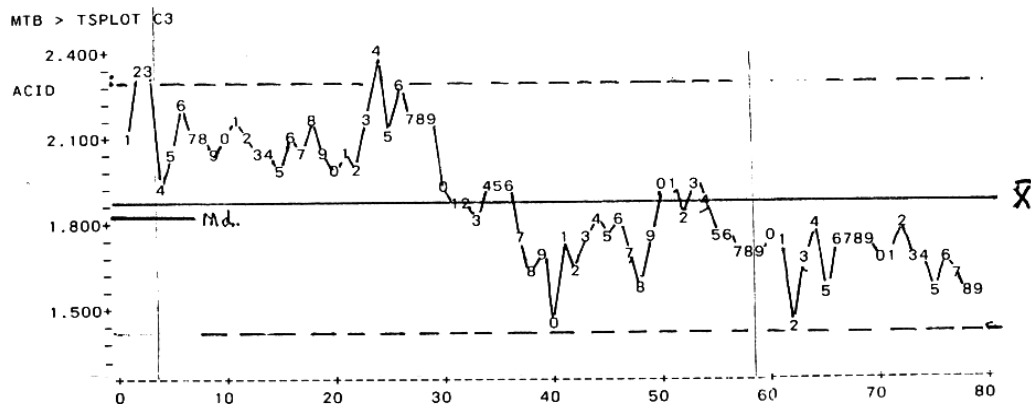


Figure 7.4d: Plot of Log Sulfate vs. Time

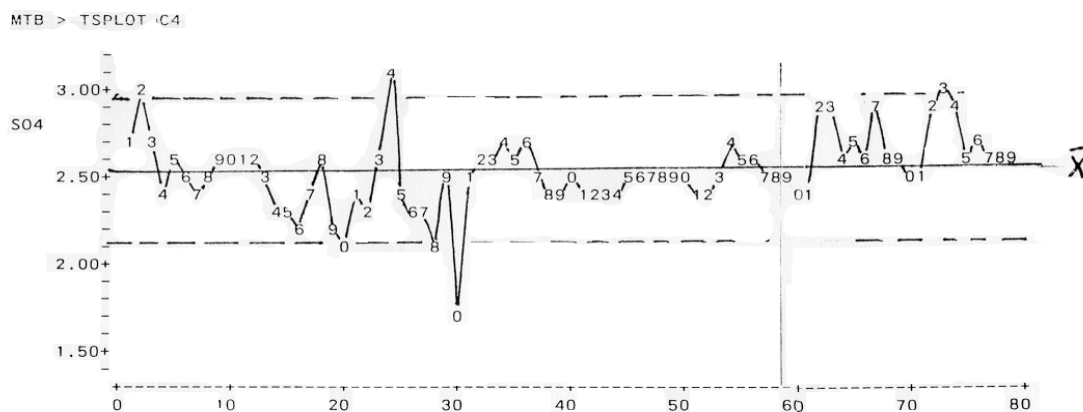
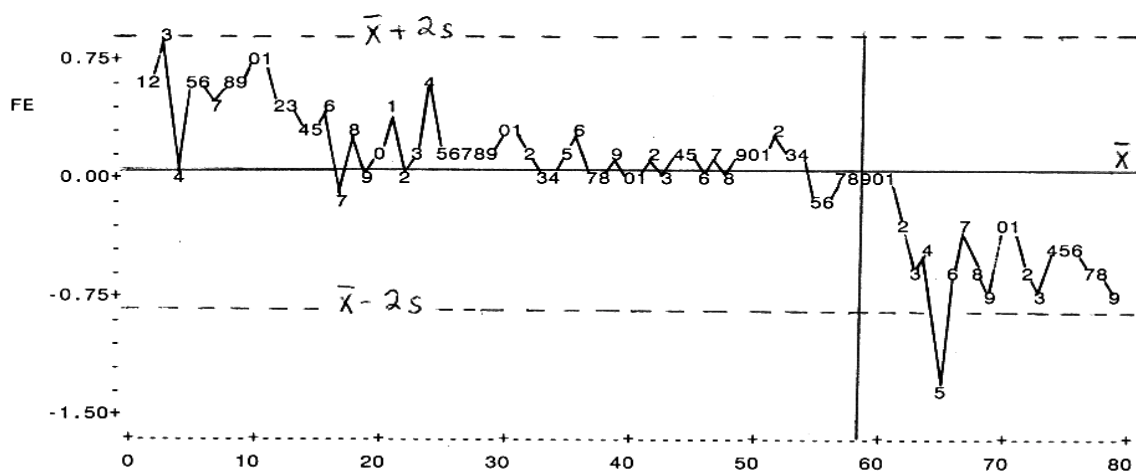
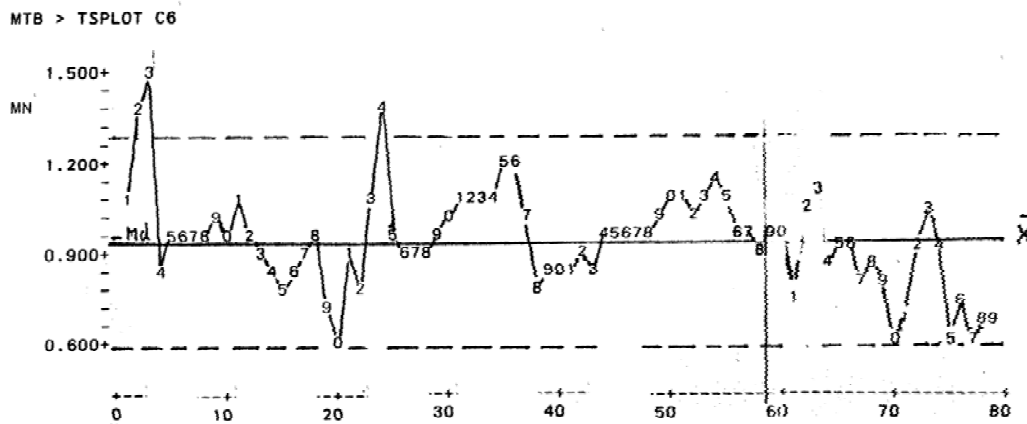


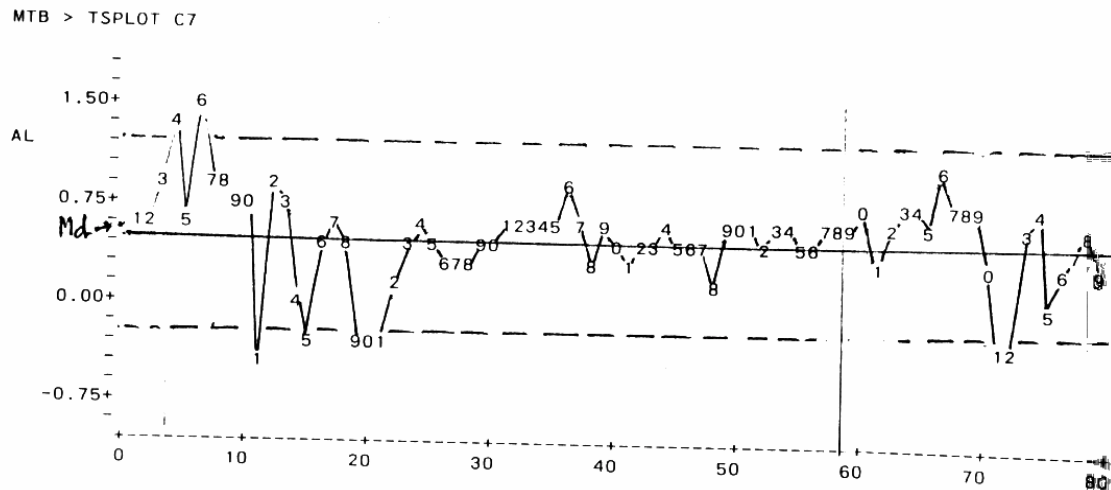
Figure 7.4e: Plot of Log Iron vs. Time



**Figure 7.4f: Plot of Log Manganese vs. Time**



**Figure 7.4g: Plot of Log Aluminum vs. Time**



### Diagnosis of Time Series Models Using Autocorrelation and Partial Autocorrelation Functions

The ACF of days or dates of observation numbers 1 through 79 (Figure 7.5a), shows a steep but uniform decline as would be expected from the consistent increase (i.e., a strong trend) present in Figure 7.4a. The corresponding Pacf (Figure 7.5b) has one large spike at lag 1. The first difference is likely to be a random walk.

The ACF of flow has no distinct patterns, with a single small spike at lag 1 (Figure 7.5c). The Pacf is similar to the ACF (Figure 7.5d), and a simple auto regression (AR(1)) or moving average (MA(1)) model would do equally well (or poorly) in describing the behavior. Acidity, on the

other hand, shows a marked decline in the Acf (Figure 7.5e) similar to that in Figure 7.5a, but somewhat less uniform. The Pacf (Figure 7.5f) has one large spike at lag 1. Here, a first difference is necessary to reduce the variation to a stationary series. Then a simple AR or MA would probably suffice. The Acf and Pacf of sulfate are very similar (Figures 7.5g and 7.5h, respectively), and resemble the corresponding graphs of flow (Figures 7.5c and 7.5d). A small spike at lag 1 and a few subdued features are not likely to be significant.

Iron shows an exponential decline in Acf (Figure 7.5i). The Pacf has a large spike at lag 1, indicating there is a trend over time which should be removed before the series becomes approximately stationary (as shown in Figure 7.5j). The other features appear to be overwhelmed by the trend.

The Acf and Pacf of variation in concentration of log manganese (Figures 7.5k and 7.5l) show similar, if slightly less distinct, characteristics as log Fe concentration. Their variations, in overall terms, are somewhat similar. Again a large spike at lag 1 requires a first difference, but the remainder of the variation is probably not significant. The variation in Acf and Pacf of aluminum (Figures 7.5m and 7.5n) are very similar to each other and to sulfate. Modeling should begin with a simple AR(1) or MA(1) and the complexity should be increased if there are any spikes which are significantly above background.

It appears evident that there are two types of variables in terms of their variation patterns. The first type is very like the first differences (e.g., Figure 7.5a) in possessing a strong trend, not always uniformly decreasing; but acidity, iron and possibly manganese all decrease over time. The second type (e.g., flow, sulfate, and aluminum) shows no well-marked trend but is much more irregular in behavior. When the trend is removed, residual variation will possibly be similar in all six variables.

Figure 7.5a: Autocorrelation Function of Days

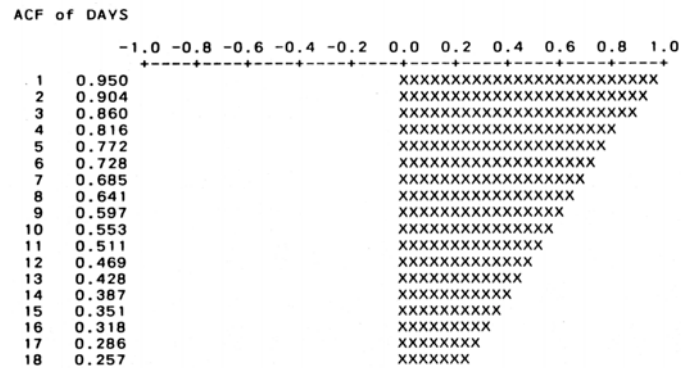


Figure 7.5b: Partial Autocorrelation Function of Days

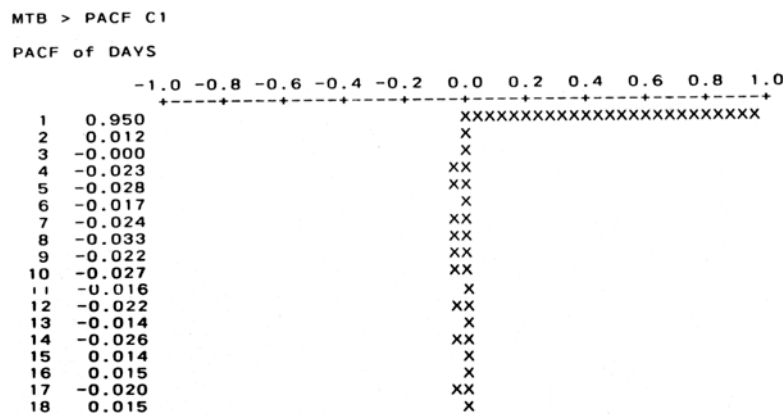


Figure 7.5c: Autocorrelation Function of Flow

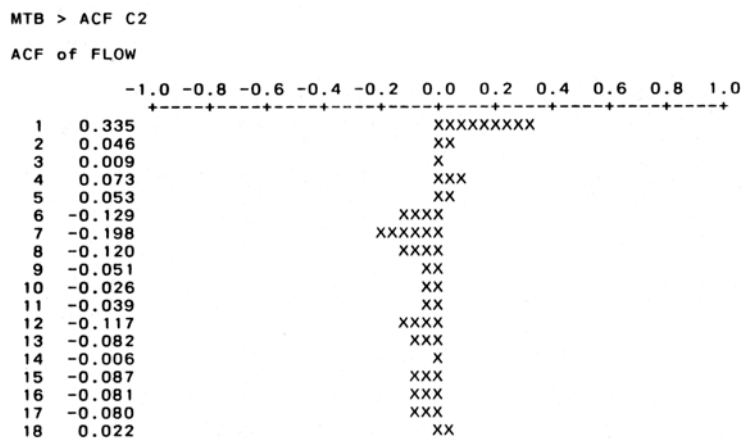


Figure 7.5d: Partial Autocorrelation Function of Flow

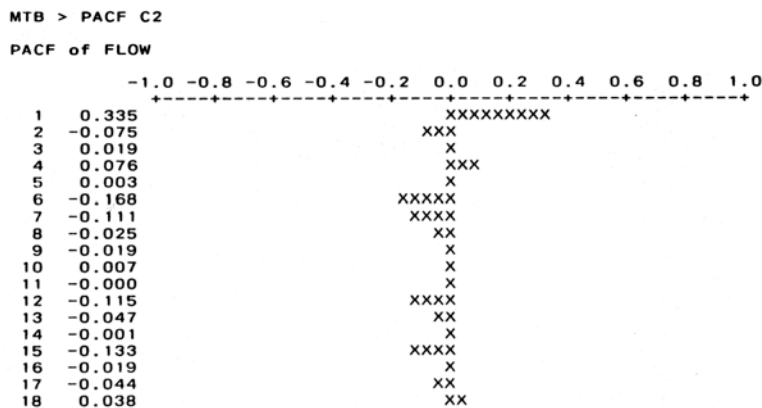


Figure 7.5e: Autocorrelation Function of Acid

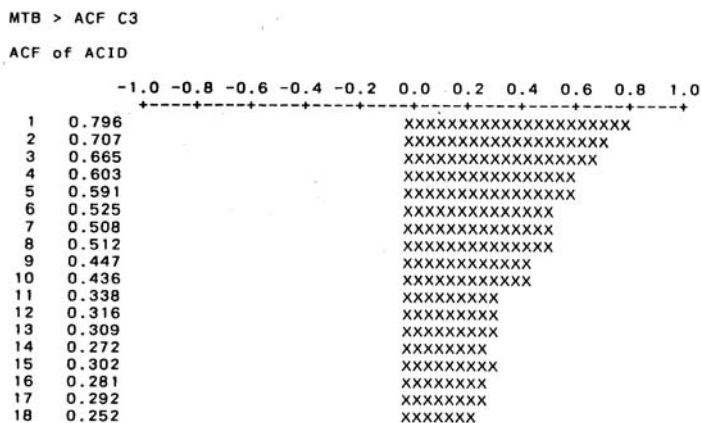
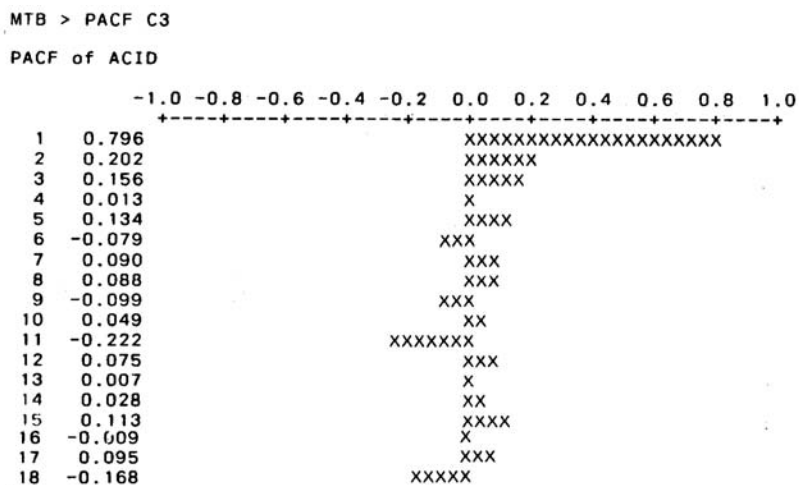
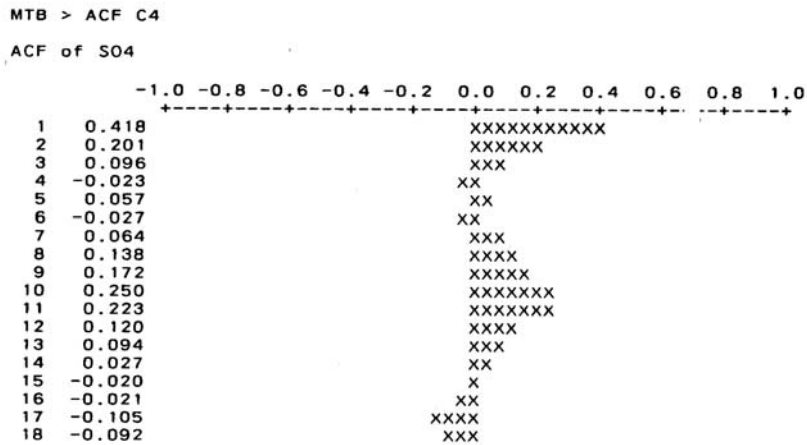


Figure 7.5f: Partial Autocorrelation Function of Acid

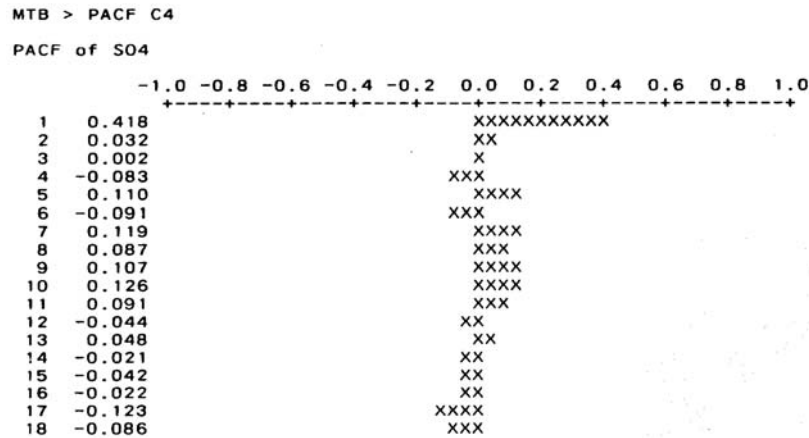




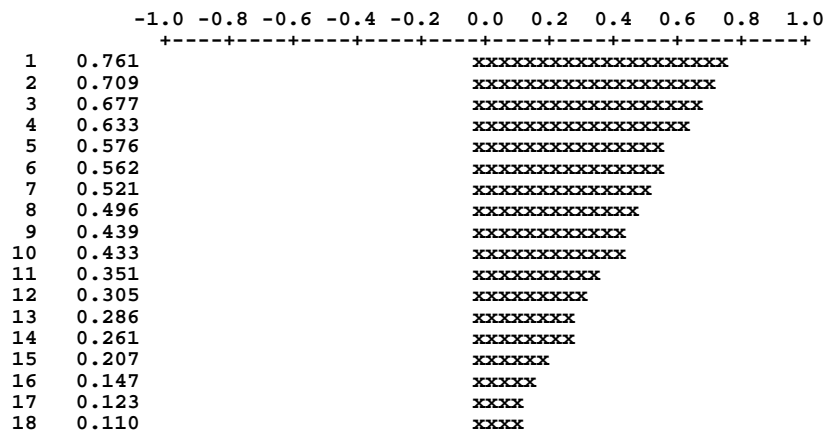
**Figure 7.5g: Autocorrelation Function of Sulfate**



**Figure 7.5h: Partial Autocorrelation Function of Sulfate**



**Figure 7.5i: Autocorrelation Function of Iron**



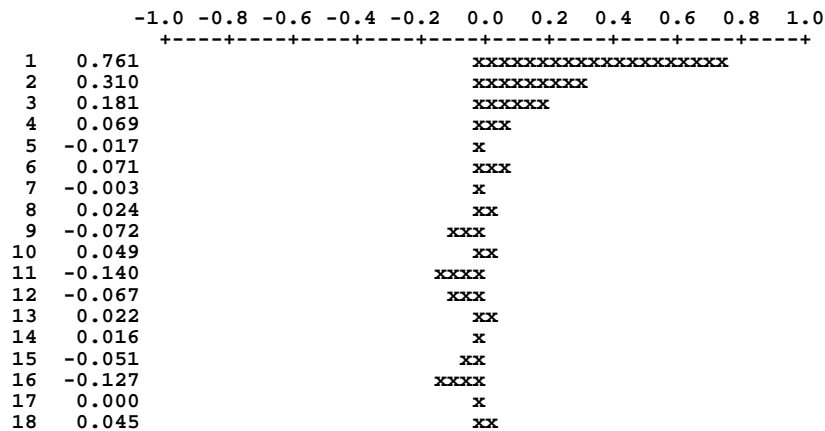
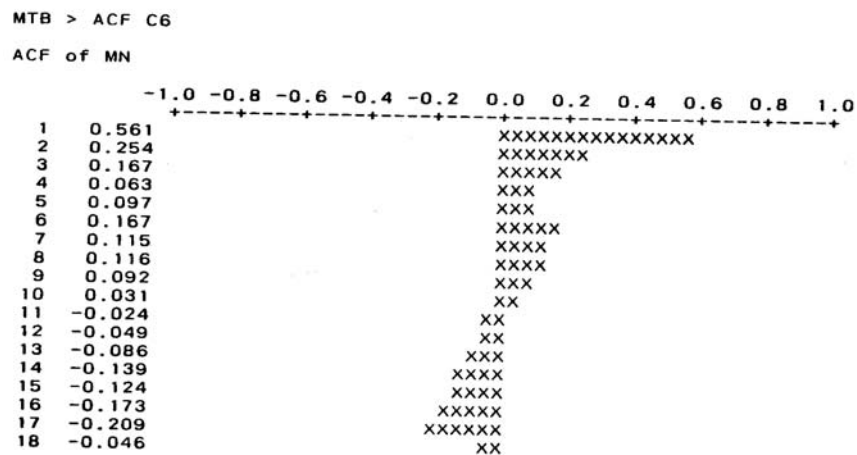
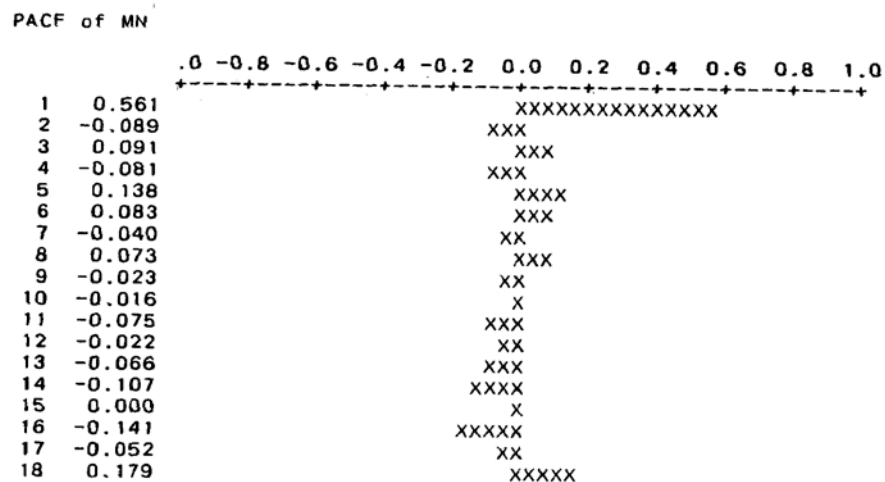
**Figure 7.5j: Partial Autocorrelation Function of Iron****Figure 7.5k: Autocorrelation Function of Manganese****Figure 7.5l: Partial Autocorrelation Function of Manganese**

Figure 7.5m: Autocorrelation Function of Aluminum

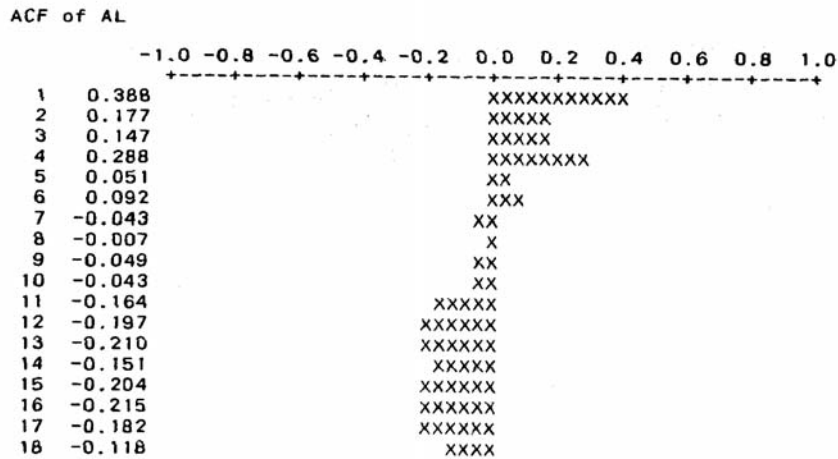
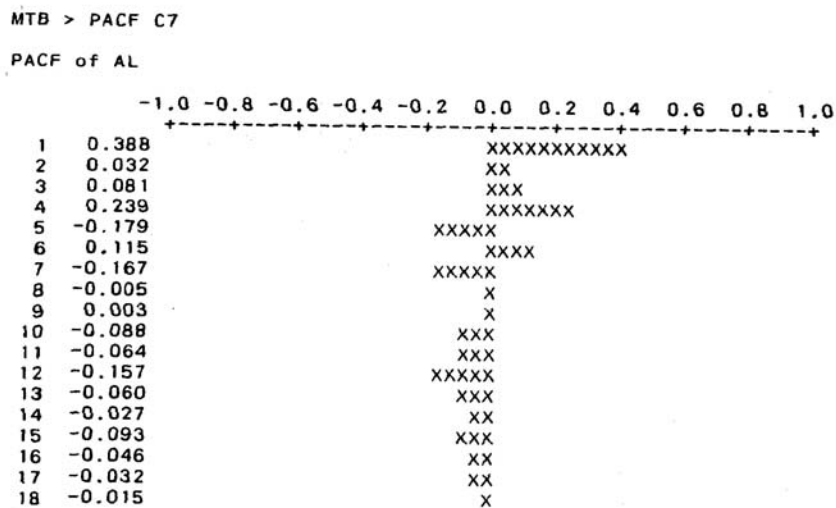


Figure 7.5n: Partial Autocorrelation Function of Aluminum



### Box-Jenkins Modeling of Variation in the Seven Variables

On the basis of the above diagnostics, variation in flow was modeled using both the AR (1,0,0) and MA (0,0,1) models. Tests of the AR (1) model outcome showed no correlation between the mean and the AR coefficient ( $\hat{\Phi}_1$ ). The residual possessed a chi-square of 9.16 with 22 degrees of freedom (df), yielding a probability of greater than 0.99 that the residual variation is white noise. Both the Acf and Pacf of the residuals were free of unusual spikes. The relationship is shown in Table 7.5. The residual standard deviation is  $\hat{\sigma}_e = 0.356$  compared with an original standard deviation of  $\hat{\sigma} = 0.373$ , a small improvement.

**Table 7.5: Equations of Models Fitted to Variables from the Fisher Deep Mine**

			$\hat{\sigma}_e$	$\hat{\sigma}$
<b>1a.</b>	Flow AR (1)	$z_t = 0.336z_{t-1} + 1.791 + a_t$	0.356	0.373
<b>1b.</b>	Flow MA (1)	$z_t = 1.788 + a_t + 0.340a_{t-1}$	0.354	
<b>2.</b>	Acid MA (1)	$z_t = a_t - 0.533a_{t-1}$	0.119	0.218
<b>3.</b>	SO <sub>4</sub> MA (1)	$z_t = 2.532 + 0.375a_{t-1} + a_t$	0.197	0.212
<b>4a.</b>	Fe AR (1)	$z_t = 0.850z_{t-1} + 0.044 + a_t$	0.252	0.411
<b>4b.</b>	Fe MA (1)	$z_t = z_{t-1} + a_t - 0.612a_{t-1}$	0.219	
<b>5.</b>	Mn MA (1)	$z_t = z_{t-1} + a_t - 0.551a_{t-1}$	0.151	0.172
<b>6.</b>	Al MA (1)	$z_t = 0.495 + a_t + 0.325a_{t-1}$	0.333	0.354

In the MA(0, 0, 1) model, there is no correlation between the mean and the moving average coefficient. The chi-square of the residuals is 8.879 with 22 df, a probability of  $P > 0.99$  against white noise. (The resulting equation is given in Table 7.5, 1b). The residual standard deviation is 0.354, which is very close to the AR value of 0.356. The models have similar equations and similar residual errors.

Variation in acidity requires a first difference. When an MA (0,1,1) model is fitted, the chi-square of the residuals equals 22.43 with 23 df. The probability that this would arise from a white noise series is  $0.50 > P > 0.30$ . The equation is presented in Table 7.5, number 2, and yields a residual standard deviation of 0.119 compared with an original standard deviation of 0.218, an almost 50% improvement.

The MA (0,0,1) model was fitted to the variation in sulfate concentration. The mean is linearly independent of the MA coefficient. Chi-square = 14.87 with 22 df, clearly showing ( $0.90 > P > 0.80$ ) that the residual variation is not different from that of white noise. The equation is shown

as No. 3 in Table 7.5 and the residual standard deviation is  $\hat{\sigma}_e = 0.197$  compared with an original standard deviation of 0.212, showing little improvement.

Two models were fitted to evaluate variation in iron concentration, an AR (1,0,0) and an MA (0,1,1). An AR(1) coefficient may be a fair approximation of the first difference in the MA model. The mean is relatively small, thus the AR(1) model is not as suitable as the MA (0,1,1) fitted to first differences mean set equal to zero. The standard deviations are 0.245 for the AR model and 0.219 for the MA model compared with  $\hat{\sigma} = 0.411$  for the original variable, an improvement of nearly 50 percent.

Since manganese varies in a manner similar to iron, the MA model was fitted to the first differences MA (0,1,1). The chi-square equals 23.82 with 23 df (or  $0.50 > P > 0.30$ , i.e., the residual variation is likely to be white noise). One significant spike at lag 4 remained in the Acf of the residuals. The equation of the MA (0,1,1) model is in Table 7.5. The standard deviation is 0.151 compared with an original standard deviation of 0.172, an improvement of only 10 percent.

Aluminum variation did not require a first difference thus the MA (0,0,1) model was fitted. Chi-square of the residuals equals 25.71 with 22 df,  $0.30 > P > 0.20$ . There is no correlation between the mean and the moving average coefficient. There is a significant spike at lag 4 as in the manganese model. The equation is given as No. 6 in Table 7.5. The standard deviation equals 0.333 compared with  $\hat{\sigma} = 0.354$  for the original series (a marginal improvement).

These variables appear to show two patterns of variation. The first pattern is simple MA(1) performance. The second pattern is a consistent trend, usually a decline, with time. This second pattern is best matched by the MA(1) model of the first differences. The effects of the trend are removed by taking first differences. In several cases, there is a significant spike at lag 4 in the Acf of the residuals. However, this single spike is not repeated and there is no seasonal effect. No further analysis was performed because the large gaps in the time between observations prevented any more rigorous analysis.

## **Quality Control**

The appropriate use of quality control (particularly in the form of confidence limits around the mean or median) is illustrated in Figures 7.2c and 7.2d. This enables comparison between pre-remining and post-remining water quality conditions and allows for differences in sample size.

The two standard deviation limits around the mean are also inserted in the time series plots (Figures 7.4b to 7.4g). These are confidence limits based on a sample size of one (i.e.,  $\bar{X} \pm 2\hat{\sigma}$ ). They do not appear to be very informative; very few observations fall outside these limits. In the pre-remining period, there is a tendency for the range (and the variance, see F-tests discussed earlier in this chapter) to be very much larger than after remining commences. As a result, all the observations after remining tend to fall within these rather wide limits.

## Summary

The most important outcome of this analysis is to show that the pattern of variation in these six variables falls into two groups. The first group (flow, sulfate, and aluminum) appears to be unaffected by remining. The second group (acidity, iron, and manganese) shows a marked improvement after remining begins. This improvement is shown in both means (medians) and variances. The means are lower and the variances less after remining began than prior to remining.